

The Common Core State Standards in Mathematics

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The Mathematics experiences in early childhood settings should concentrate on (1) number (which includes whole number, operations, and relations) and (2) geometry, spatial relations, and measurement, with more mathematics learning time devoted to number than to other topics. (*Mathematics Learning in Early Childhood: Paths Toward Excellence and Equity*, NRC, 2009)

Coherence

... articulated over time as a sequence of topics and performances that are logical and reflect, where appropriate, the sequential or hierarchical nature of the disciplinary content from which the subject matter derives. That is, what and how students are taught should reflect not only the topics that fall within a certain academic discipline, but **also the key ideas** that determine how knowledge is organized and generated within that discipline. This implies that to be coherent, a set of content standards must evolve from particulars (e.g., the meaning and operations of whole numbers, including simple math facts and routine computational procedures associated with whole numbers and fractions) to deeper structures inherent in the discipline. (Schmidt, Cogan, Houang)

Counting & Cardinality						Ratios & Proportional Relationships		
Operations and Algebraic Thinking						The Number System		
Number and Operations in Base Ten						Expressions and Equations		
			Fractions					Functions
Measurement and Data								
Geometry						Geometry		
						Statistics and Probability		
K	1	2	3	4	5	6	7	8

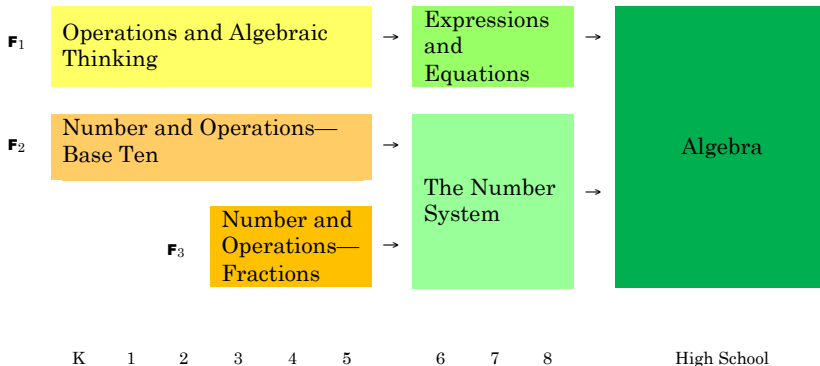
High School

- Number and Quantity
 - ...
- Algebra
 - Seeing Structure in Expressions
 - Arithmetic with Polynomials and Rational Expressions
 - Creating Equations
 - Reasoning with Equations and Inequalities
- Functions
 - Interpreting Functions
 - Building Functions
 - Linear, Quadratic, and Exponential Models
 - Trigonometric Functions
- Modeling
 - ...
- Geometry
 - ...
- Statistics and Probability
 - ...

Practice standards

- Make sense of problems and persevere in solving them
- Reason abstractly and quantitatively
- Construct viable arguments and critique the reasoning of others
- Model with mathematics
- Use appropriate tools strategically
- Attend to precision
- Look for and make use of structure
- Look for and express regularity in repeated reasoning

Flows in the standards



Ties in the standards

	Operations and Algebraic Thinking	Number and Operations in Base Ten	Fractions
1	Understand and apply properties of operations and the relationship between addition and subtraction.	Use place value understanding and properties of operations to add and subtract.	
2		Use place value understanding and properties of operations to add and subtract.	
3	Understand properties of multiplication and the relationship between multiplication and division.	Use place value understanding and properties of operations to perform multi-digit arithmetic.	
4		Use place value understanding and properties of operations to perform multi-digit arithmetic.	Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.
5			Apply and extend previous understandings of multiplication and division to multiply and divide fractions.

Graded ramp up to Algebra in Grade 8

Grades K–7: Properties of operations, similarity, ratio and proportional relationships, rational number system.

Grade 8: Focus on linear equations and functions

- Expressions and Equations
 - Work with radicals and integer exponents.
 - Understand the connections between proportional relationships, lines, and linear equations.
 - Analyze and solve linear equations and pairs of simultaneous linear equations.
- Functions
 - Define, evaluate, and compare functions.
 - Use functions to model relationships between quantities.

Fraction clusters, Grades 3–6

Grade 3

- Develop understanding of fractions as numbers.

Grade 4

- Extend understanding of fraction equivalence and ordering.
- Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.
- Understand decimal notation for fractions, and compare decimal fractions.

Grade 5

- Use equivalent fractions as a strategy to add and subtract fractions.
- Apply and extend previous understandings of multiplication and division to multiply and divide fractions.

Grade 6

- Apply and extend previous understandings of multiplication and division to divide fractions by fractions.
- Compute fluently with multi-digit numbers and find common factors and multiples.
- Apply and extend previous understandings of numbers to the system of rational numbers.

Sample Fraction Standard, Grade 4

4.NF.4 Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.

- (a) Understand a fraction a/b as a multiple of $1/b$. *For example, use a visual fraction model to represent $5/4$ as the product $5 \times (1/4)$, recording the conclusion by the equation $5/4 = 5 \times (1/4)$.*
- (b) Understand a multiple of a/b as a multiple of $1/b$, and use this understanding to multiply a fraction by a whole number. *For example, use a visual fraction model to express $3 \times (2/5)$ as $6 \times (1/5)$, recognizing this product as $6/5$. (In general, $n \times (a/b) = (n \times a)/b$.)*
- (c) Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. *For example, if each person at a party will eat $3/8$ of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?*

Develop understanding of statistical variability

- Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers. For example, “How old am I?” is not a statistical question, but “How old are the students in my school?” is a statistical question because one anticipates variability in students’ ages.
- Understand that a set of data collected to answer a statistical question has a distribution which can be described by its center, spread, and overall shape.
- Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number.

Functional thinking stream

- Grade 3** Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations.
- Grade 4** Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself.
- Grade 5** Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane.
- Grade 6** Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and

Functional thinking stream (continued)

Grade 7 Recognize and represent proportional relationships between quantities.

- (a) Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.
- (b) Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.
- (c) Represent proportional relationships by equations. *For example, if total cost t is proportional to the number n of items purchased at a constant price p , the relationship between the total cost and the number of items can be expressed as $t = pn$.*
- (d) Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points $(0, 0)$ and $(1, r)$ where r is the unit rate.

Middle school foundations

- Hands-on experience with transformations.
- Low tech (transparencies) or high tech (dynamic geometry software).

High school rigor and applications

- Properties of rotations, reflections, translations, and dilations are assumed, proofs start from there.
- Connections with algebra and modeling

- Progressions
(<http://ime.math.arizona.edu/progressions/>)
- Illustrative Mathematics Project
(<http://illustrativemathematics.org>).
- Technical manual
- NCTM-AMTE-NCSM-ASSM task force (see description at <http://commoncoretools.wordpress.com>).
- Project headed by Bill Bush to develop criteria for judging textbooks.
- NCTM sample tasks for reasoning and sense-making,
<http://www.nctm.org/hsfocus>
- My blog, Tools for the Common Core,
<http://commoncoretools.wordpress.com>